Image Theory

Vertical Electric Dipole (VED)

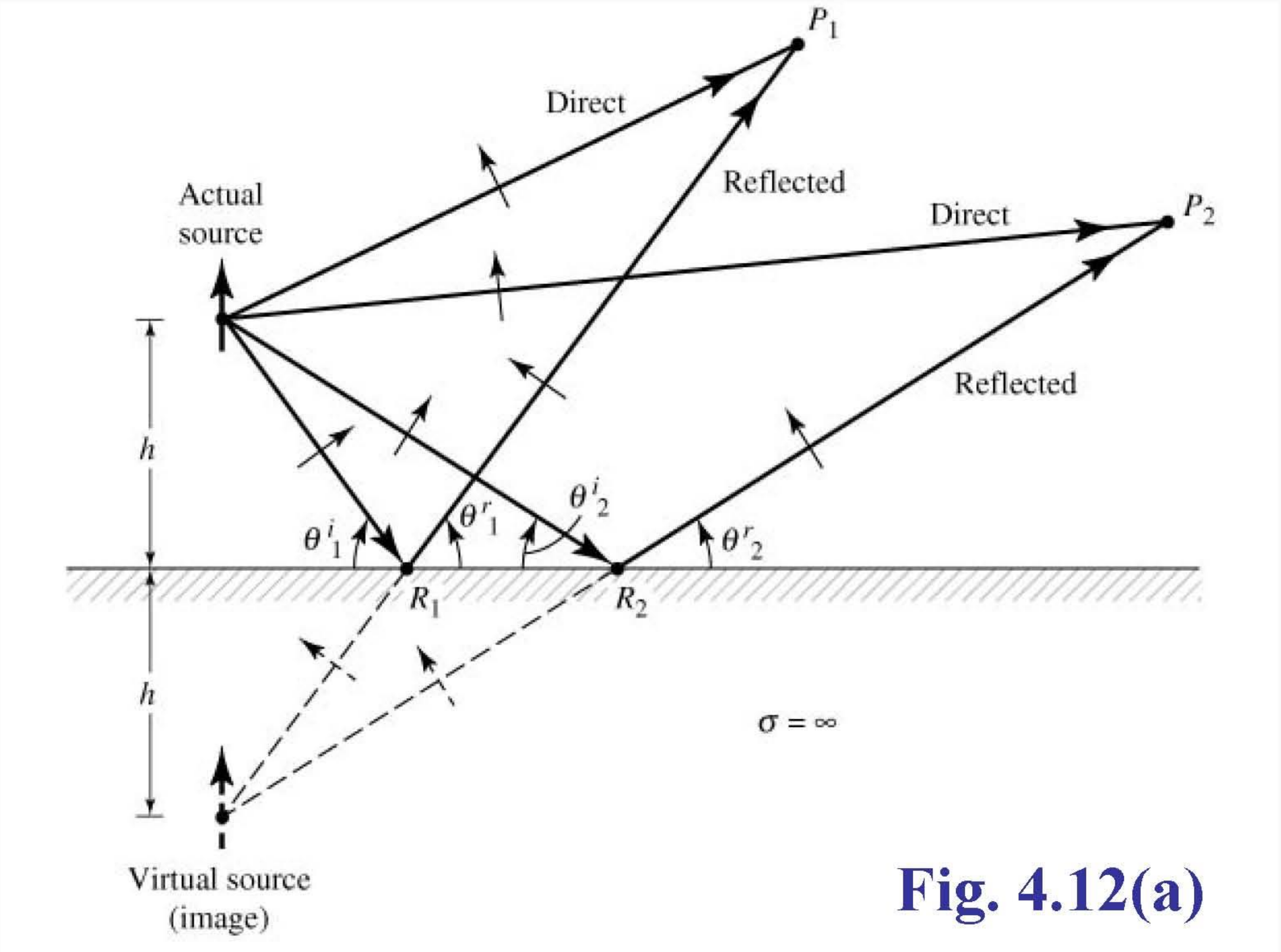


Fig. 4.12(a)

Electric Conductor

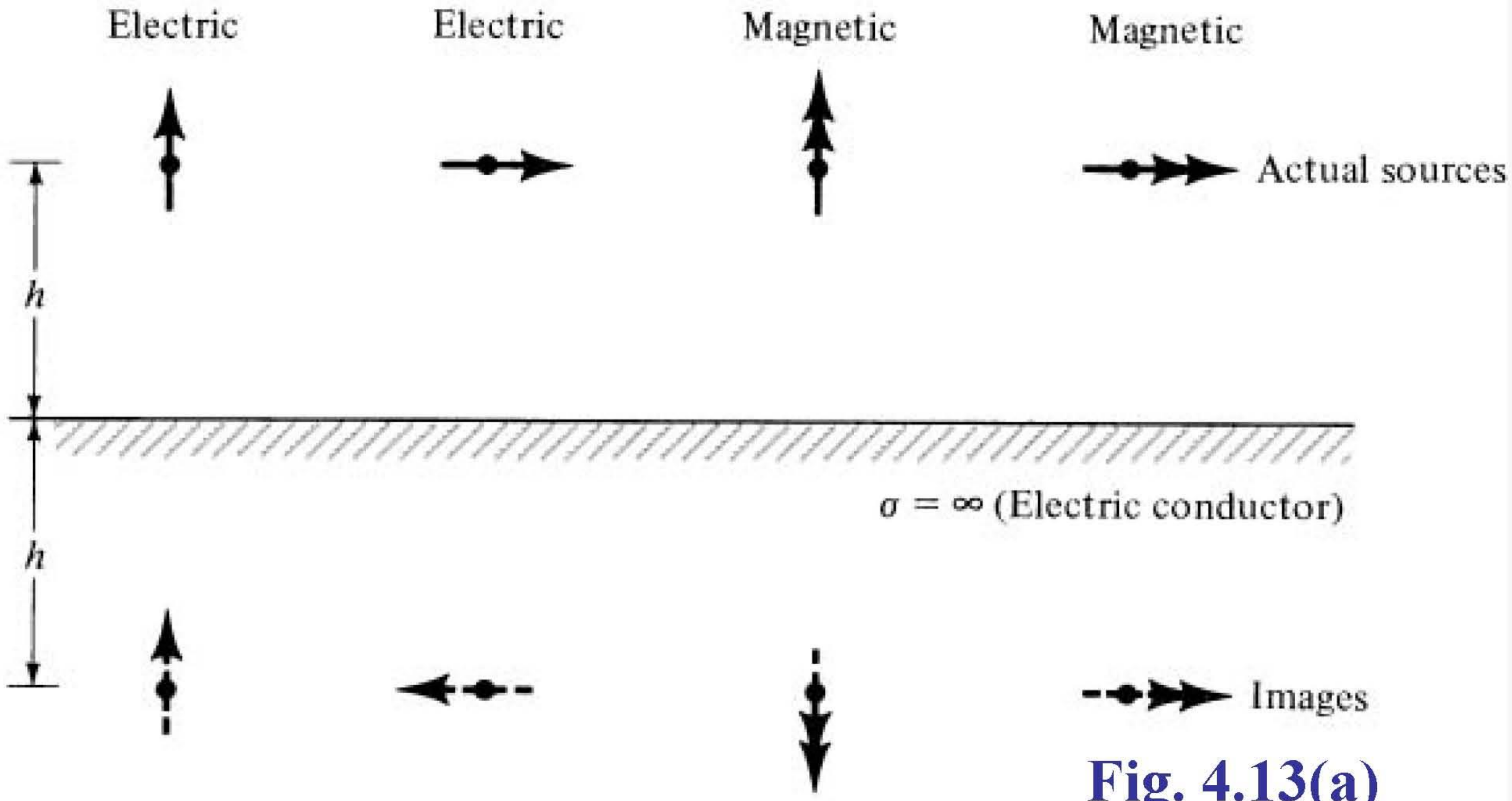


Fig. 4.13(a)

Magnetic Conductor

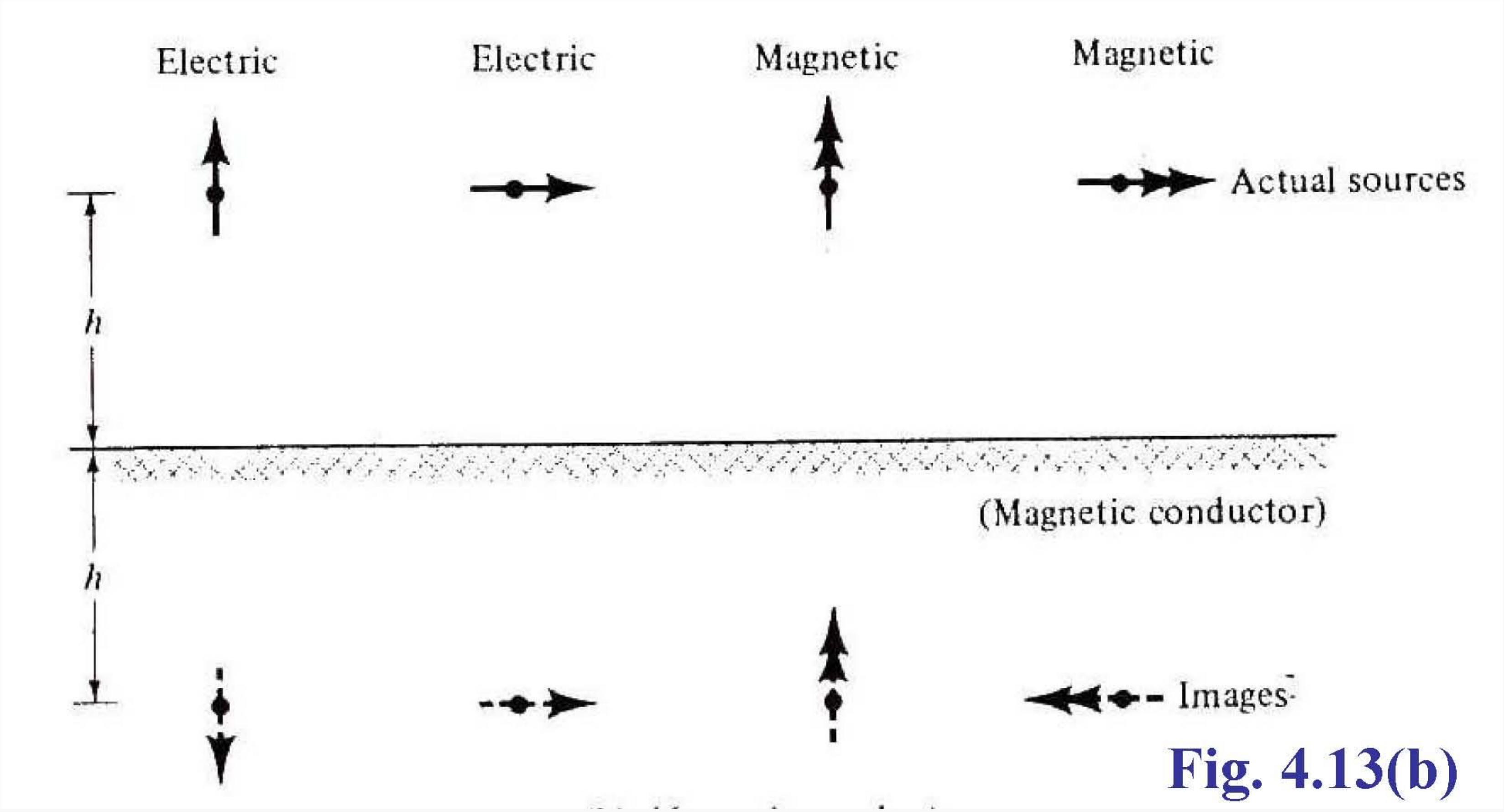


Fig. 4.13(b)

$$E_\theta^d = j\eta \frac{kI_o \ell e^{-jkr_1}}{4\pi r_1} \sin\theta_1 \qquad (4\text{-}94)$$

$$E_\theta^r = R_v \left\{ j\eta \frac{kI_o \ell e^{-jkr_2}}{4\pi r_2} \sin\theta_2 \right\} \qquad (4\text{-}95)$$

$$= +1 \left\{ j\eta \frac{kI_o e^{-jkr_2}}{4\pi r_2} \sin\theta_2 \right\} \qquad (4\text{-}95a)$$

$$E_\theta^t = E_\theta^d + E_\theta^r$$

Vertical Electric Dipole above Infinite Perfect Electric Conductor (PEC)

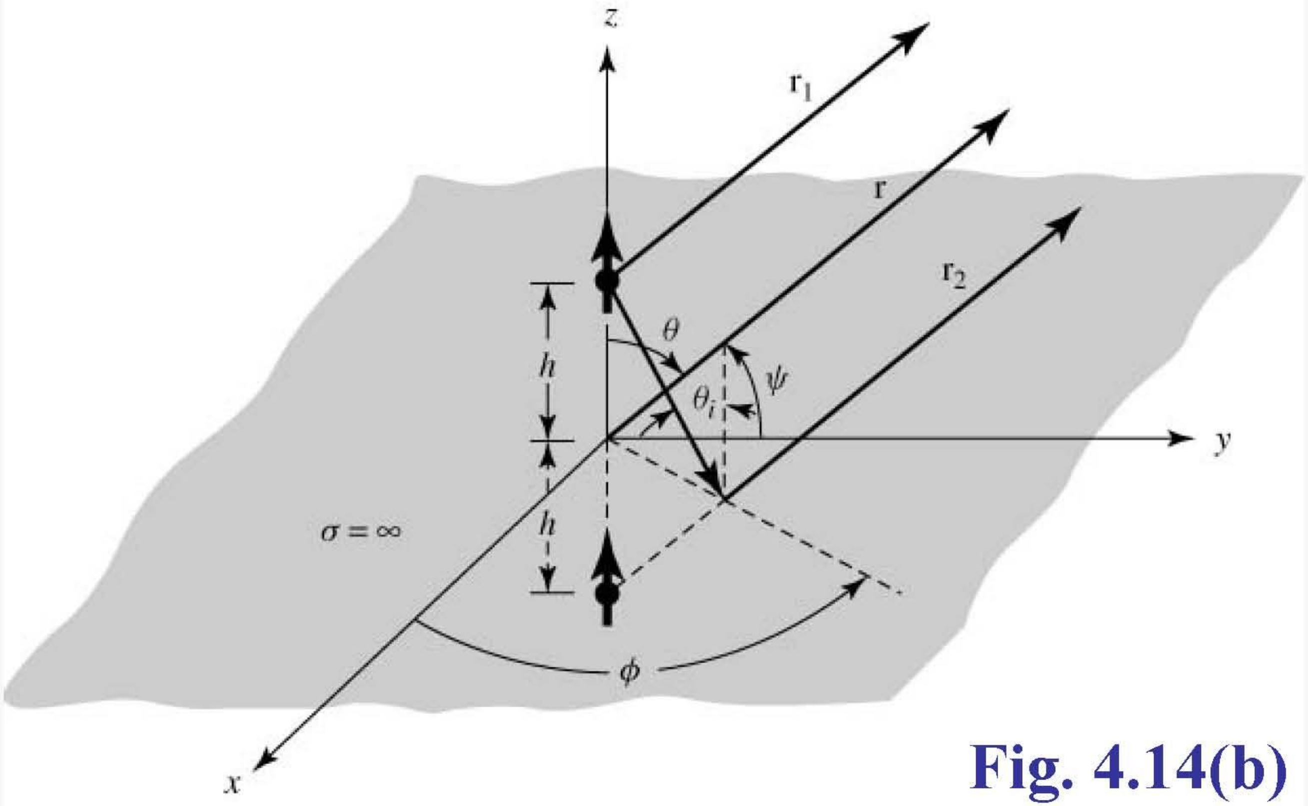


Fig. 4.14(b)

Far-Field Approximations

$$\left.\begin{aligned} r_1 &= r - h\cos\theta \\ r_2 &= r + h\cos\theta \end{aligned}\right\} \text{ for phase terms} \qquad (4\text{-}97a,b)$$

$$r_1 \cong r_2 \cong r \; \} \text{ for amplitude terms} \qquad (4\text{-}98)$$

$$\theta_1 \cong \theta_2 \cong \theta$$

$$E_\theta = \underbrace{j\eta \frac{k I_o \ell e^{-jkr}}{4\pi r} \sin\theta}_{Element \quad Factor} \; \underbrace{\{2\cos(kh\cos\theta)\}}_{Array \quad Factor}$$

$z \geq 0$

$$E_\theta = 0$$

$z < 0$

(4-99)

Vertical Dipole

$$\text{Number of Lobes} \cong 2\left(\frac{h}{\lambda}\right)+1 \qquad (4\text{-}100)$$

$$h >> \lambda$$

Directivity and Radiation Resistance of Vertical Element Above a Ground Plane

$$D_0 = \frac{2}{\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]}$$

$$R_r = 2\pi\eta\left(\frac{l}{\lambda}\right)^2\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]$$

Maximum Directivity Occurs When:

$$kh = 2.881$$

$$h = \frac{2.881}{k} = \frac{2.881}{2\pi / \lambda} = 0.4585\lambda$$

$$D_o = 6.566 = 8.173(dB)$$

Monopoles and Dipoles

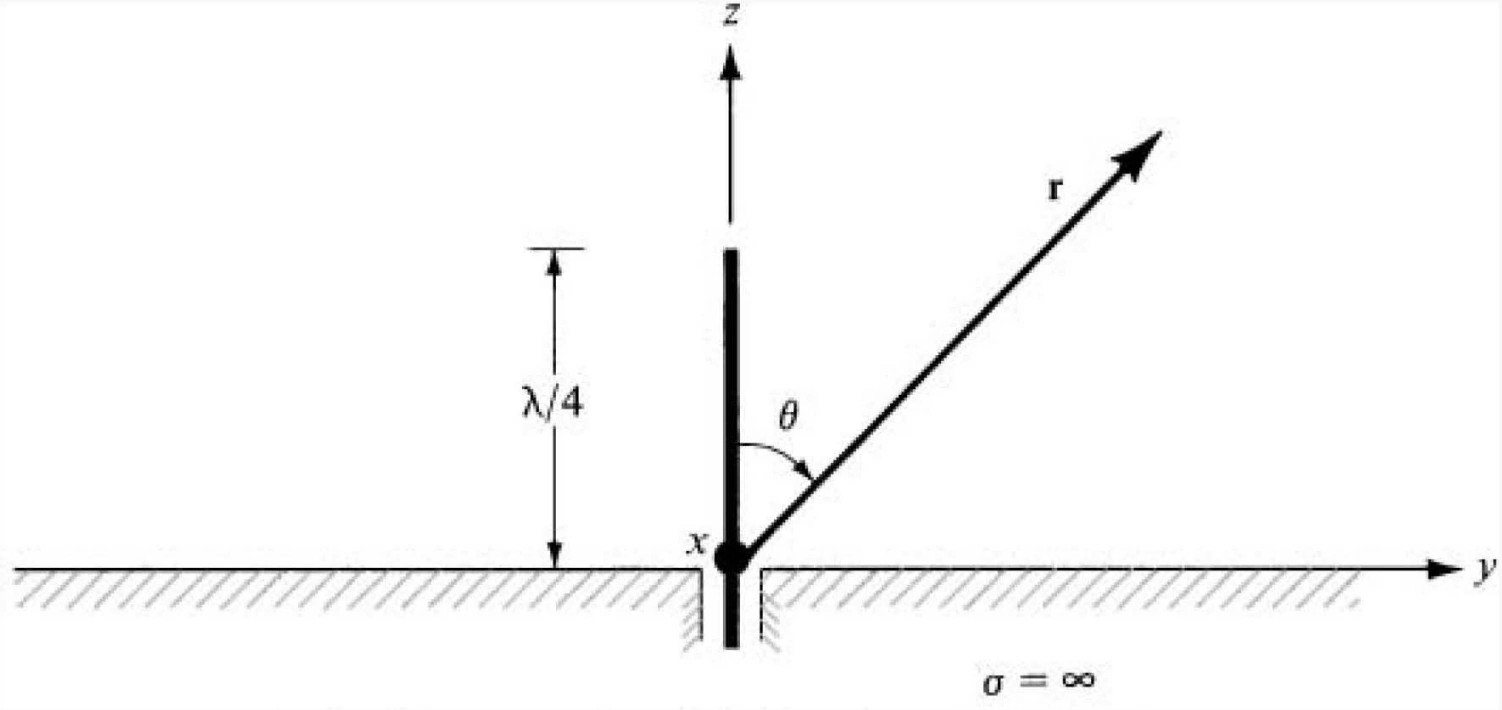
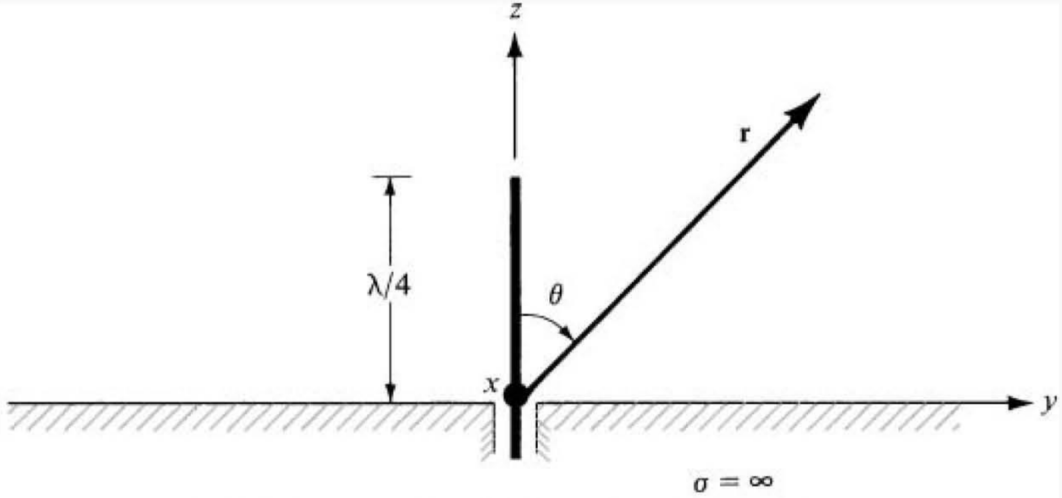


Fig. 4.19(a)

Monopoles and Dipoles

$$
\begin{aligned}
Z_{in} \text{ (monopole) } &= \frac{1}{2} Z_{in} \text{ (dipole)} \\
D_0 \text{ (monopole) } &= 2 D_0 \text{ (dipole)}
\end{aligned}
\tag{4-106}
$$

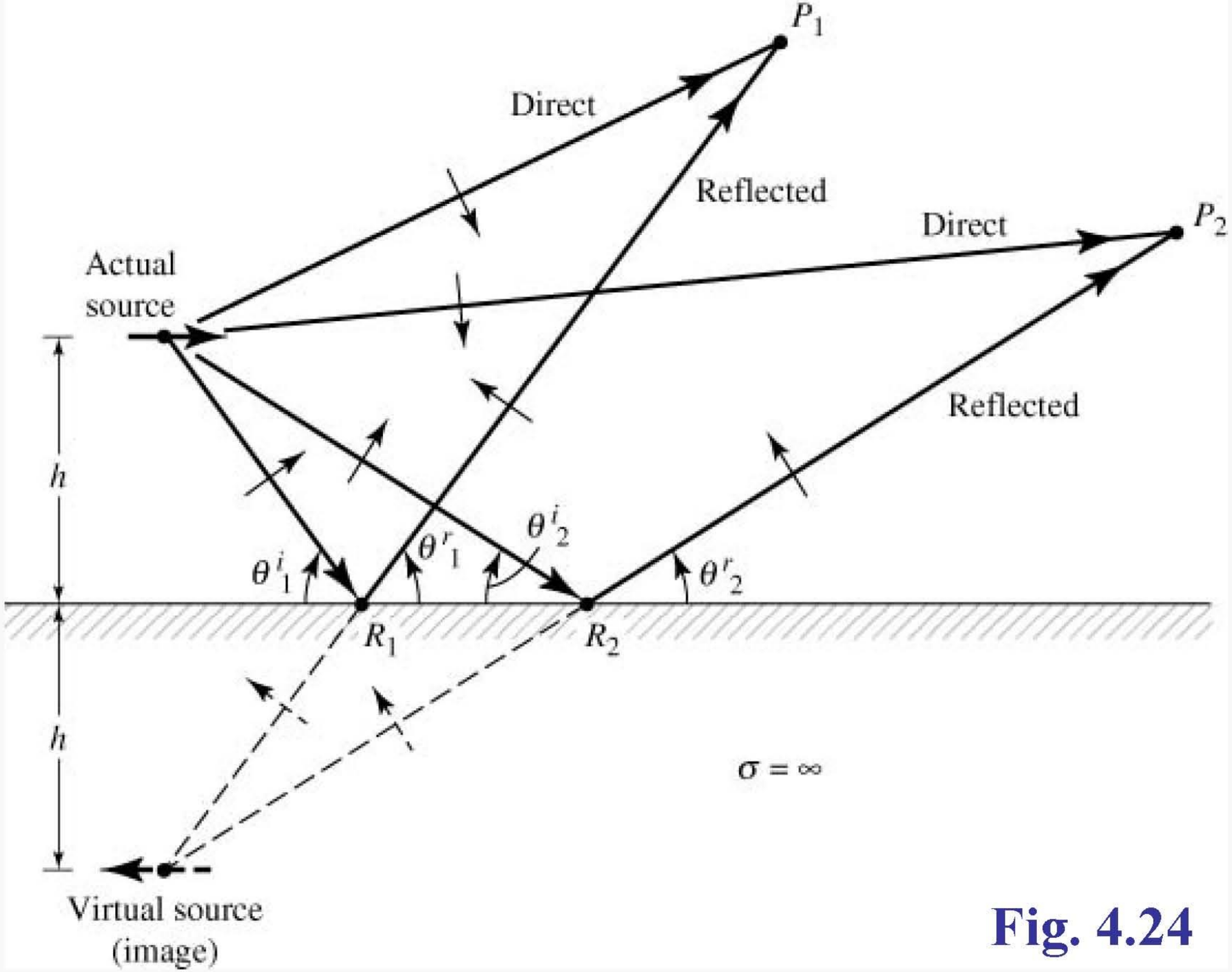
$\lambda/4$ Monopole on Infinite Electric Conductor



(a) $\lambda/4$ monopole on infinite electric conductor

$$Z_{in} = 36.5 + j21.25$$

Fig. 4.19(a)



P_1
Direct
Reflected
P_2
Direct
Actual
source
Reflected
h
θ^i_1
θ^r_1
θ^i_2
θ^r_2
R_1
R_2
h
$\sigma = \infty$
Virtual source
(image)

Fig. 4.24

Horizontal Electric Dipole Above an Infinite Perfect Electric Conductor

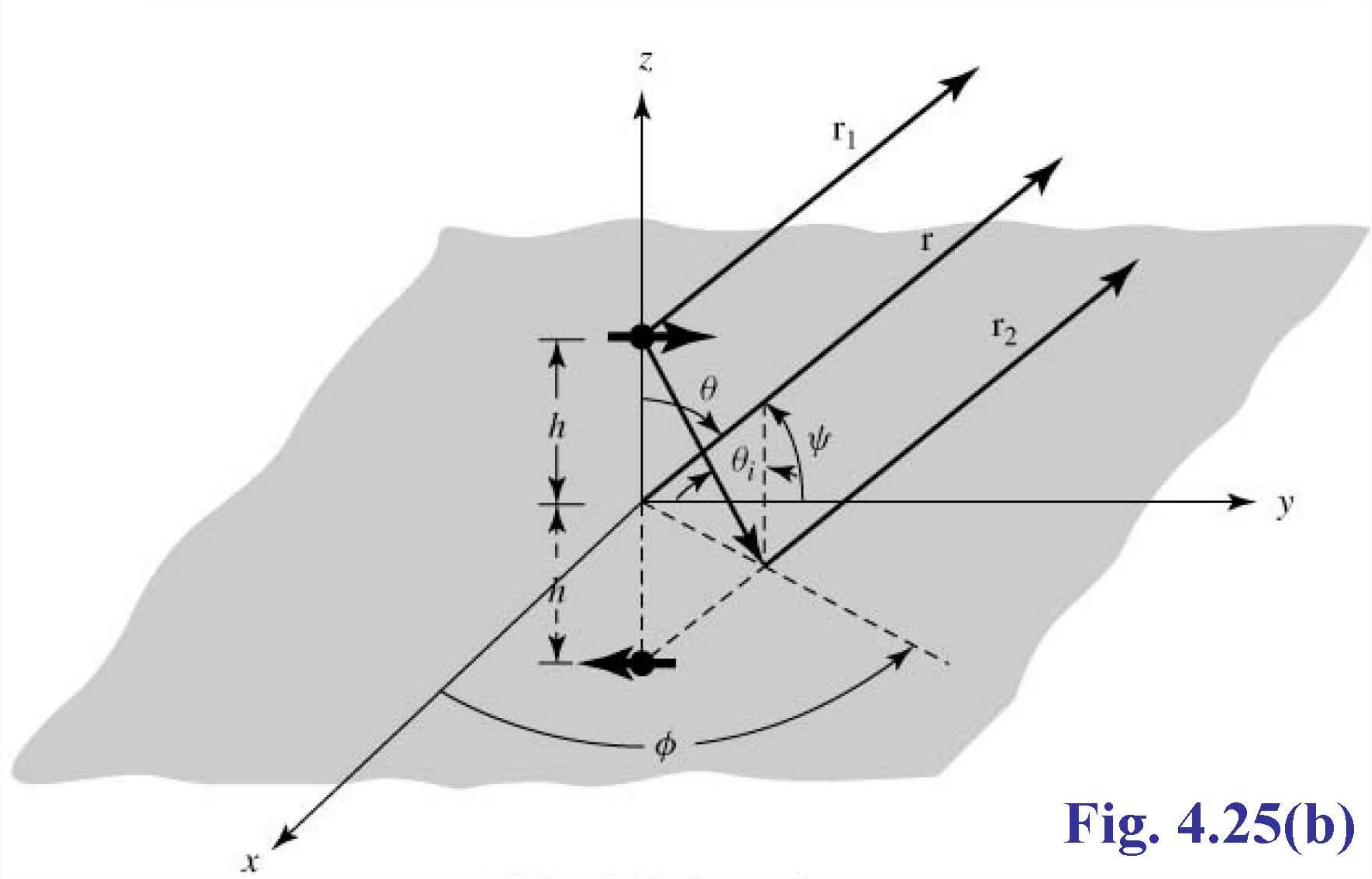


Fig. 4.25(b)

$$E_\psi^d = j\eta \frac{kI_o \ell e^{-jkr_1}}{4\pi r_1} \sin\psi \quad (4\text{-}111)$$

$$E_\psi^r = jR_h \eta \frac{kI_o \ell e^{-jkr_2}}{4\pi r_2} \sin\psi \quad (4\text{-}112)$$

$$= -j\eta \frac{kI_o \ell e^{-jkr_2}}{4\pi r_2} \sin\psi \quad (4\text{-}112a)$$

Far-Field Approximations

$$\left.\begin{aligned} r_1 &\cong r - h\cos\theta \\ r_2 &\cong r + h\cos\theta \end{aligned}\right\}$$ for phase terms (4-115a)

$$\left. r_1 \cong r_2 \cong r \right\}$$ for amplitude terms (4-115b)

$$\theta_1 \cong \theta_2 \cong \theta$$

$$E_\psi^t = E_\psi^d + E_\psi^r = \underbrace{j\eta \frac{kI_o \ell e^{-jkr}}{4\pi r}\sqrt{1-\sin^2\theta\sin^2\phi}}_{Element\ \ Pattern}\ \underbrace{\left[2j\sin(kh\cos\theta)\right]}_{Array\ \ Factor}$$

(4-116)

Elevation Plane ($\phi = 90^o$) Amplitude Patterns

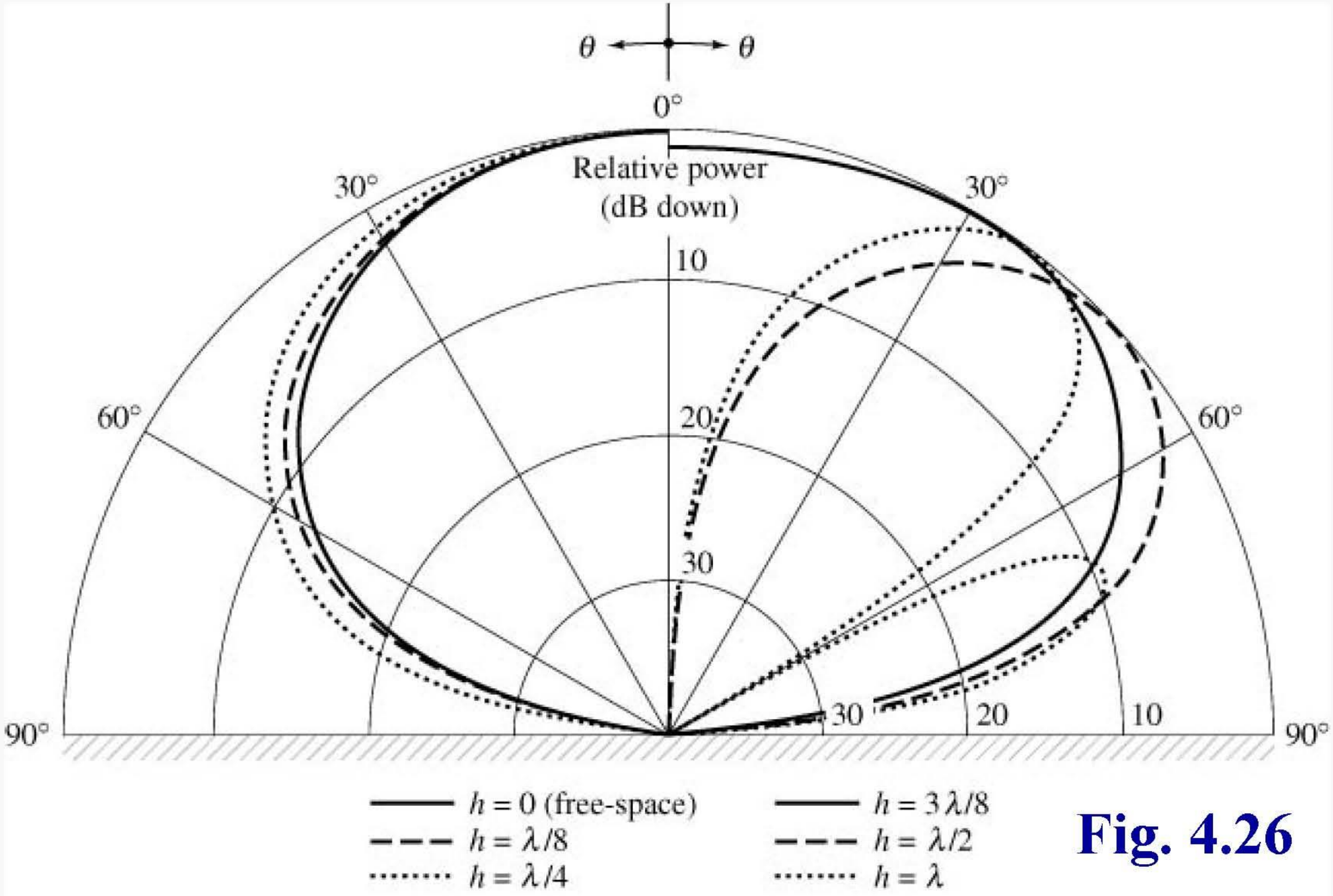


Fig. 4.26

Horizontal Dipole

$$\text{Number of Lobes} \cong 2\left(\frac{h}{\lambda}\right) \tag{4-117}$$

$$h >> \lambda$$

Radiation Resistance of Horizontal Element above a Ground Plane

$$R_r = \frac{2P_{rad}}{|I_0|^2}$$

$$= \eta\pi \left(\frac{l}{\lambda}\right)^2 \left[\frac{2}{3} - \frac{\sin(2kh)}{2kh} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]$$

(4-119)

Directivity of Horizontal Element above a Ground Plane

$$D_0 = \begin{cases} \dfrac{4\sin^2(kh)}{\left[\dfrac{2}{3} - \dfrac{\sin(2kh)}{2kh} - \dfrac{\cos(2kh)}{(2kh)^2} + \dfrac{\sin(2kh)}{(2kh)^3}\right]}, & h \le \dfrac{\lambda}{4} \\ \dfrac{4}{\left[\dfrac{2}{3} - \dfrac{\sin(2kh)}{2kh} - \dfrac{\cos(2kh)}{(2kh)^2} + \dfrac{\sin(2kh)}{(2kh)^3}\right]}, & h > \dfrac{\lambda}{4} \end{cases}$$

Lossy Surface

- Antenna characteristics (especially radiation efficiency) at LF and MF (below 3 MHz) are profoundly and adversely affected by the lossy earth.

Vertical Polarization

$$E_\theta = j\eta \frac{kI_o \ell e^{-jkr}}{4\pi r} \sin\theta \left[e^{jkh\cos\theta} + R_v e^{-jkh\cos\theta} \right]$$

$$R_v = \frac{\eta_0 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_0 \cos\theta_i + \eta_1 \cos\theta_t} = -R_{\parallel} \qquad \text{(4-125)}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}}, \qquad \eta_0 = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$\gamma_0 \sin\theta_i = \gamma_1 \sin\theta_t \qquad \text{(4-126)}$$

$$\gamma_0 = j\beta_0$$

$$\gamma_1 = \sqrt{j\omega\mu_1\left(\sigma_1 + j\omega\varepsilon_1\right)} = \alpha_1 + j\beta_1$$

1- For PEC ground plane, $R_v = 1$, this causes pattern maximum direction at horizon (desired direction).

2- For non-PEC ground plane, $R_v \neq 1$, this causes pattern maximum direction is shifted away from horizon (undesired direction).

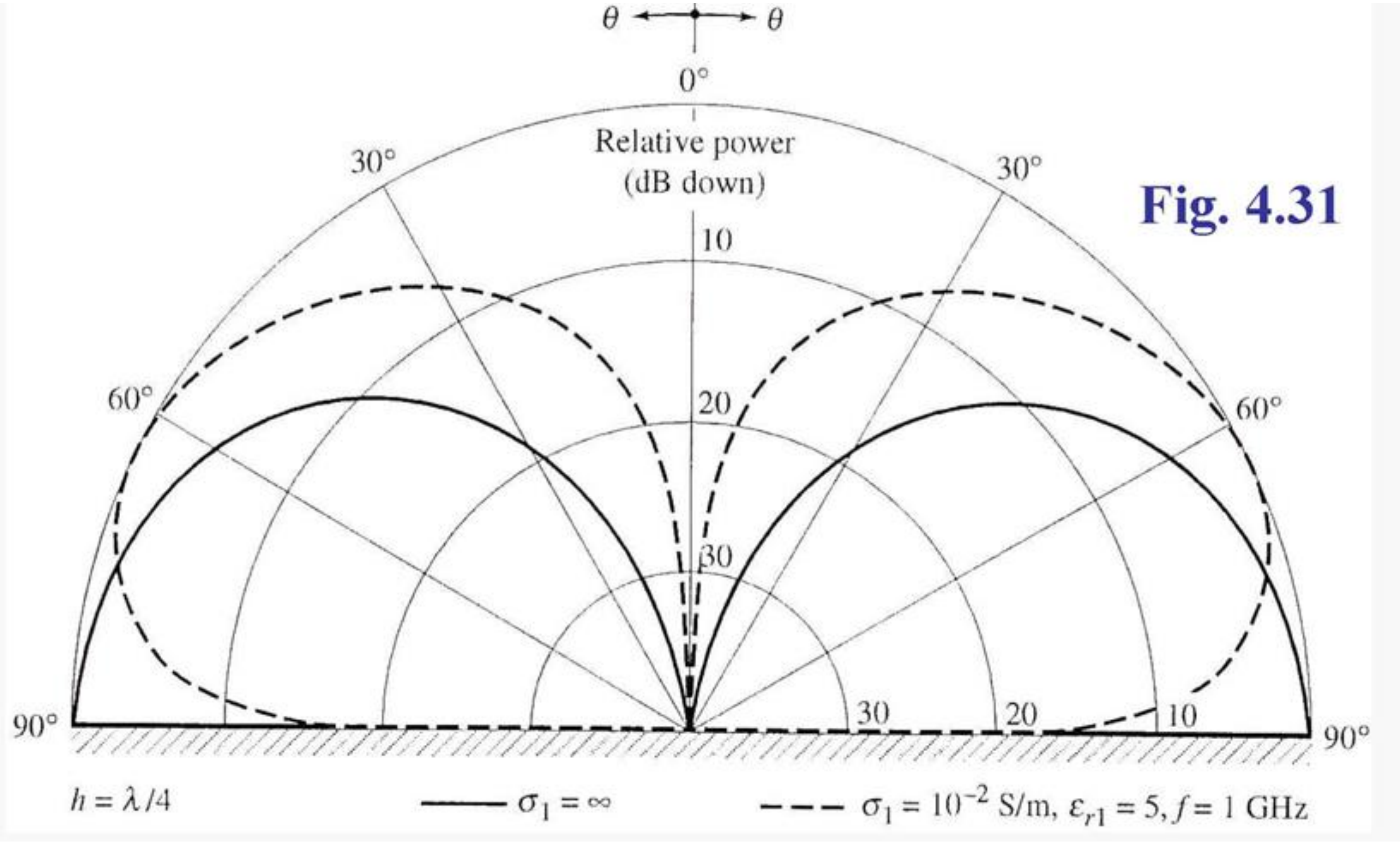
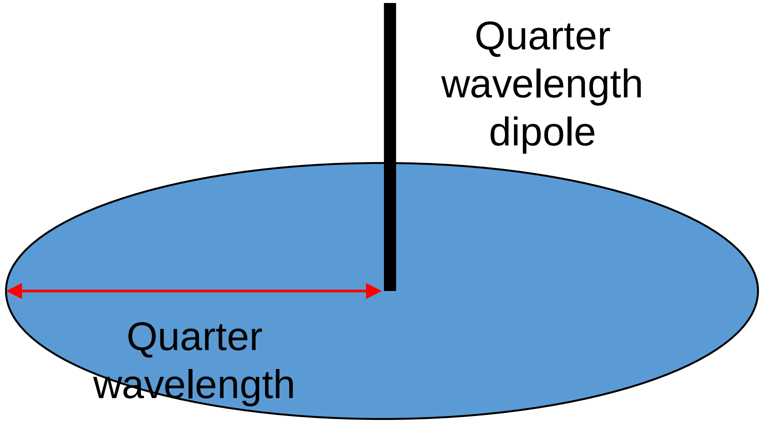


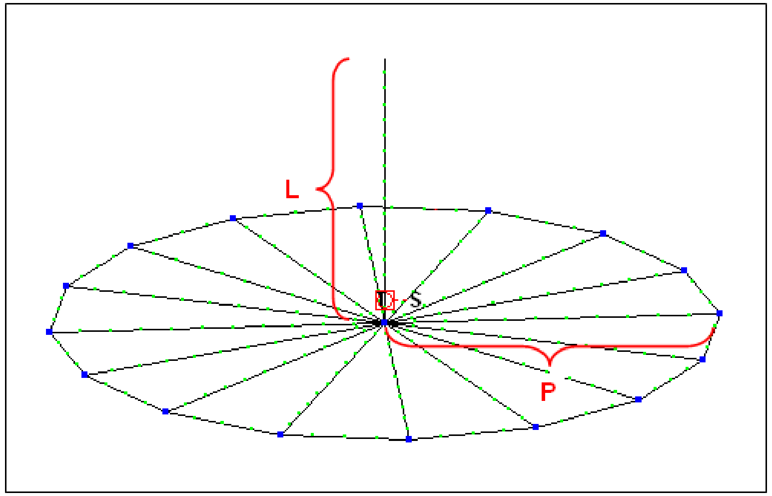
Fig. 4.31

Effect of Imperfectly Conducting, Flat Earth

- To improve the radiation efficiency at these frequencies, radial wires or metallic disks are sometimes placed on the ground to simulate a perfectly conducting ground plane.



Metallic disc



Radial wires

Horizontal Polarization

$$E_{\psi} = j\eta \frac{kI_o \ell e^{-jkr}}{4\pi r} \sqrt{1 - \sin^2\theta \sin^2\phi} \left[e^{jkh\cos\theta} + R_h e^{-jkh\cos\theta} \right]$$

(4-129)

$$R_h = \begin{cases} R_{\perp} & for \quad \phi = 0^o, \ 180^o \quad plane \\ R_{\parallel} & for \quad \phi = 90^o, 270^o \quad plane \end{cases}$$

(4-128)

$$R_{\parallel} = \frac{\eta_1 \cos\theta_t - \eta_0 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_0 \cos\theta_i}$$

(4-125)

$$R_{\perp} = \frac{\eta_1 \cos\theta_i - \eta_0 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_0 \cos\theta_t}$$

(4-128a)

Horizontal Polarization

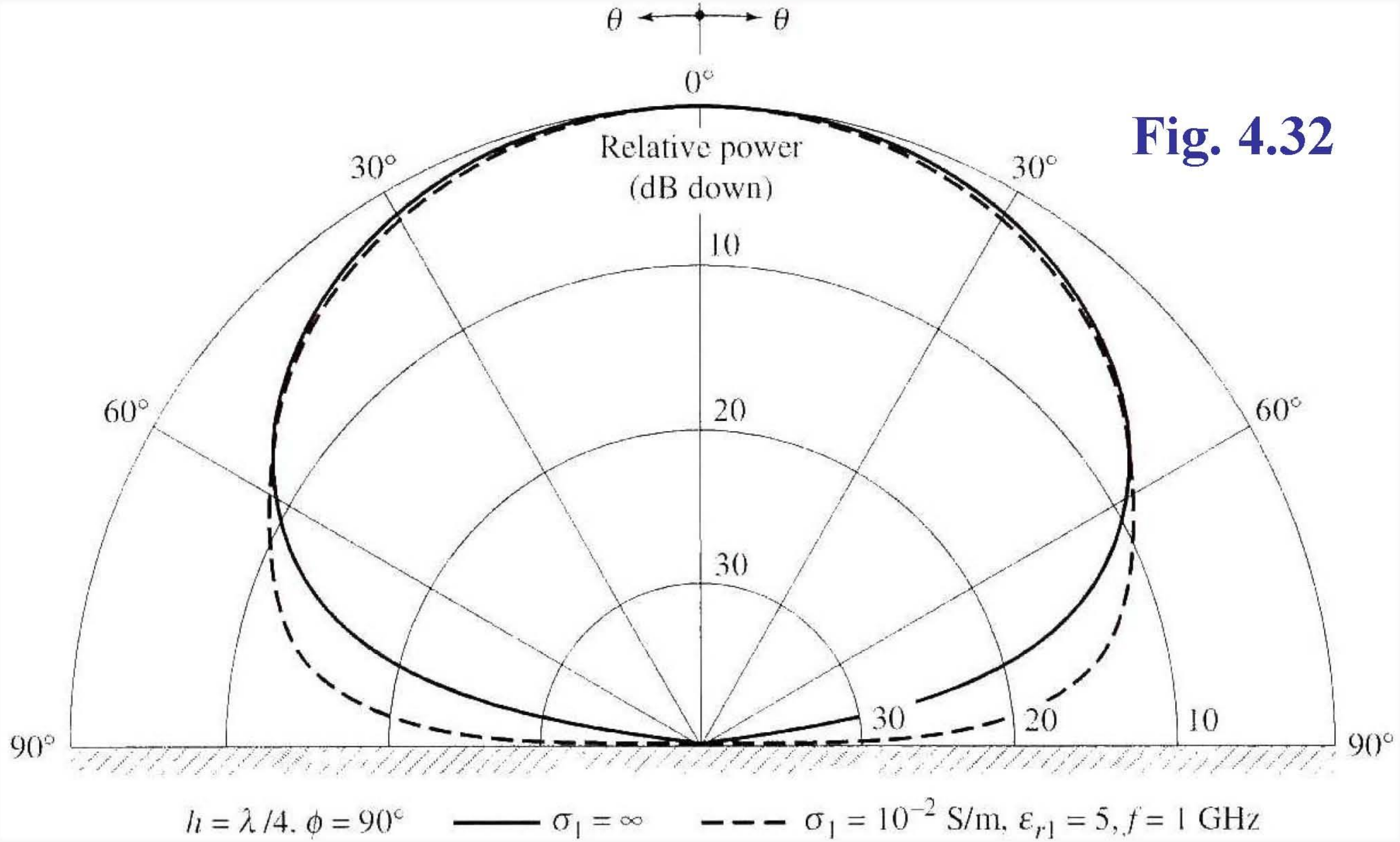


Fig. 4.32

Recommended for sky-wave communications